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AN EXPERIMENTAL STUDY OF THE EFFECTIVE THERMAL CONDUCTIVITY OF A SHEARED SUSPENSION OF RIGID SPHERES

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Abstract—An experimental study was conducted to measure the effective thermal conductivity of a sheared suspension of rigid spherical particles. The objective was to verify the theoretical prediction of Leal (1973) for a dilute suspension undergoing shear at low particle Peclet number, and to extend the range of the experiments to conditions beyond the scope of the theory. Surprisingly, reasonable agreement with the theoretical prediction was observed even for suspensions of moderate concentrations (volume fraction $\varphi \le 0.25$) and higher Peclet numbers [$Pe \sim O(1)$]. The trend of the data, however, verifies the obvious fact that the theory does not completely describe the transport behavior at higher concentrations and Peclet numbers. The range of quantitative applicability of Leal's result is apparently only for $Pe < 0.01$ and $\varphi < 0.01$, but the changes in the effective thermal conductivity in this domain were too small to be measured in our apparatus.

I. INTRODUCTION

A large number of theoretical and experimental studies have been undertaken to investigate the rheological properties of suspensions. The constitutive behavior of suspensions for transport of heat or molecular species in the presence of a nonuniform bulk flow has, on the other hand, been largely neglected even though it is of great importance in many engineering applications. The present paper reports an experimental study of the effect of a simple shear flow upon heat transfer by conduction in a suspension of rigid, spherical particles.

The pioneering work on the effective thermal properties of a suspension was published over 100 years ago by Maxwell (1873) who obtained a prediction for the effective thermal conductivity for a dilute *stationary* dispersion of solid spheres. Following this analysis, fairly extensive theoretical studies were also made for stationary suspensions of both high and low concentrations and with both spherical and nonspherical particles (Jeffrey 1973; Rocha & Acrivos 1973; Jeffrey 1974). Since the macroscopic conductive heat flux is simply the ensemble average of the microscale conduction flux, the effective conductivity of a *stationary* suspension differs from that of the suspending fluid only when the conductivities of the two phases are different. When the suspension undergoes a nonuniform bulk flow, however, a local disturbance flow is induced in the vicinity of each particle, thus creating a local convective flux of heat. This additional local convective flux, dependent on the type and strength of the bulk flow, is known to contribute to the effective bulk conductivity of the suspension (Leal 1973). However, because of the structural complexities of flowing suspensions, detailed theoretical predictions of their thermal properties have been limited to dilute suspensions, simple shear flow and linear bulk temperature fields. The first rigorous analysis was due to Leal (1973) who considered the case of a sheared dilute suspension of spherical drops in the limit of low particle Peclet number, i.e. $\mathrm{Pe} \equiv (a^2 G/\kappa) \ll 1$, where a is the drop radius, G the shear rate, and κ the thermal diffusivity ($\equiv k/\rho C_p$). McMillen & Leal later considered a dilute suspension of : (1) slightly deformed drops (1975): and (2) a dilute suspension of rigid prolate spheroidal particles with rotational Brownian motion (1979), both for low particle Peclet numbers and simple shear flow. Nir & Acrivos (1976) also studied the problem of rigid spherical particles in a linear shear flow with no Brownian motion, but in the limit of large local Peclet number.

Existing experimental studies (Singh 1968; Collingham 1968) are largely qualitative in nature, aimed more at establishing the existence of an augmented thermal conductivity due to local convection at the particulate level, than at providing a detailed relationship between the bulk thermal conductivity and shear rate. An initial attempt to quantitatively measure the effective thermal conductivity of a sheared suspension of rigid spherical particles was made by McMillen (1976) in this laboratory, using a rotating Couette device with a heated inner cylinder and a cooled outer cylinder. However, McMillen's original equipment lacked sufficient sensitivity and accuracy to provide quantitative data. The present work is a direct continuation of that begun by McMillen. However, by incorporating various modifications to improve the experiment, we have been able to obtain quantitatively accurate results. We consider conditions of small concentration and small local Peclet number for a suspension of rigid spherical particles in an attempt to compare with the available theoretical predictions. In addition, we have extended the study to higher concentrations and intermediate Peclet numbers which have not been considered in the existing theories.

2. THEORETICAL BACKGROUND

Prior to describing the experiments, it is useful to briefly review the theoretical analyses which were mentioned in the previous section for a suspension of rigid spheres. These theories consist of asymptotic limiting results for very large and very small values of the local particle Peclet number, respectively.

The small Peclet number case was considered by Leal (1973) who calculated the effective transverse conductivity for a dilute suspension of equal-sized spherical drops. The effective conductivity, k_s, evaluated for a rigid spherical particle was predicted to be

$$k_s = k_1\left\{1 + \varphi\frac{3(k_2 - k_1)}{k_2 + 2k_1} + \right.$$

$$\left.\left[1.176\frac{(k_2 - k_1)^2}{(k_2 + 2k_1)^2} + 3.0 - 0.14\frac{(k_2 - k_1)}{k_2 + 2k_1}\right] \cdot \mathrm{Pe}_1^{3/2}\varphi + 0(\mathrm{Pe}_1^2)\right\}. \quad [1]$$

Here,

$$\mathrm{Pe}_1 = \frac{a^2\rho C_p G}{k_1},$$

is the local Peclet number, while k_2 and k_1 are the thermal conductivities of the particle and suspending fluid, respectivelv. It can be seen that the effective conductivity of the suspension is predicted to always increase when the bulk shear rate is increased. The flow-induced enhancement of the transport rate is $0(\mathrm{Pe}_1^{3/2})$.

Nir & Acrivos (1976) analyzed the opposite limit of large Peclet number, again for rigid spherical particles in a linear shear flow. Because of the existence of closed stream surfaces near the particle, the calculation of the microscale temperature distribution around each particle is extremely complicated in this case and was not solved completely. By using certain assumptions about the integrability of the solution, however, Nir & Acrivos were able to provide an estimate of the flow-induced contribution to the effective conductivity,

$$\frac{k_s}{k_1} = 0(1) + A_1\varphi \mathrm{Pe}_1^{1/11}. \quad [2]$$

Here, A_1 is a constant to be determined (in principle) from the temperature distribution around each particle. The primary conclusion to be drawn from [2] is that Pe_1 must be *exceedingly* large before the asymptotically dominant $0(\mathrm{Pe}_1^{1/11})$ term becomes significant. Thus, it may be expected that k_s/k_1 will be 0(1) and nearly constant for all but the largest, $\geqslant 10^{11}$, values of the local Peclet number.

An experiment in heat transfer will generally allow values of Pe_1 in the range $Pe_1 \leq 0(1)$. For small Pe_1 and small φ, Leal's expression [1] should be applicable and one goal of the experiments was to test this postulate. In addition, we considered larger values of Pe_1 and φ, where no theory presently exists.

Several limitations of Leal's theory should be mentioned since these influenced the design of the experiments. First, the particles were assumed to be identical in size and exactly spherical (deviations from sphericity were shown by McMillen & Leal (1975) to lead to an equation for k_s of fundamentally different form from [1]). Second, the particles were assumed to be small compared to the overall dimensions of the gap so that the suspension can be viewed macroscopically as a homogeneous continuum. Provided this condition is satisfied, the particle size only influences the degree of enhancement of the heat transfer rate by its effect on the local Peclet number. For larger particles, the interaction between particle and wall becomes important and the effect of shear on the rate of heat transfer will be specific to the details of the particular apparatus. The present investigation was thus restricted to small spherical particles. Three different size particles were used in order to provide the widest possible range of values for the Peclet number, but all were much smaller than the gap width in the Couette device so that "wall" effects could be ignored in the analysis of data. Finally, the theory assumes that the material parameters, such as the viccosity and conductivity, are all constant (i.e. independent of spatial position) and this necessitates small temperature differences across the sample. Small temperature differences are also necessary in order to minimize any natural convection contributions to the macroscopic rate of heat transfer in the Couette device, which would otherwise greatly complicate the data analysis.

3. EXPERIMENTAL APPARATUS AND PROCEDURE

The Couette flow device used in this study operated with a rotating inner cylinder and a concentric, stationary outer cylinder. A temperature difference was generated across the gap by heating the inner cylinder while simultaneously cooling the outer cylinder. Natural convection was minimized by using a small gap width and by operating with a small temperature difference between the two cylinders.

The main features of the flow apparatus are shown in figure 1. The inner cylinder was machined from a hollow tube of aluminium to an outside diameter of 9.866 ± 0.0013 cm. Two high-purity Nickel-A wires of 0.05 cm dia., insulated with a 0.0025 cm coating of H-ml enamel, were wrapped tightly around a continuous groove machined in the surface of the aluminium cylinder to serve as heaters. The wires were then covered with an epoxy resin and the "heater" section machined to the final diameter. Connections of the wires from the rotating inner cylinder to the external electronics were made through four mercury cups mounted on the shaft of the cylinder (figure 1). A central heater of length 10 cm comprised the active test section in the experiment, while outer heater sections of length 5 cm were used as "guard" heating to minimize axial heat losses. By monitoring the electrical resistance of the central wire heater, the temperature of the wire could be determined, effectively using the wire as a simple resistance thermometer.

The outer, stationary cylinder was made from a precision-bore glass tube with a wall thickness of 0.3 cm, an inside diameter of 11.887 ± 0.008 cm, and a length of about 30 cm. Because the alignment of the two concentric cylinders was critical to avoid secondary flows and other complications, two carefully machined alignment rings were used in the assembly of the apparatus, and the ends of the glass cylinder were ground square by hand. Another glass cylinder with an internal diameter of approximately 20.0 cm was positioned external to the precision bore glass tube and water at a fixed temperature was circulated from a constant temperature bath into the region between the two glass cylinders. Stainless steel baffles were used to insure turbulence and a virtually constant temperature in this region.

The required electrical circuits for the test and gaurd heaters are illustrated in figure 2. For

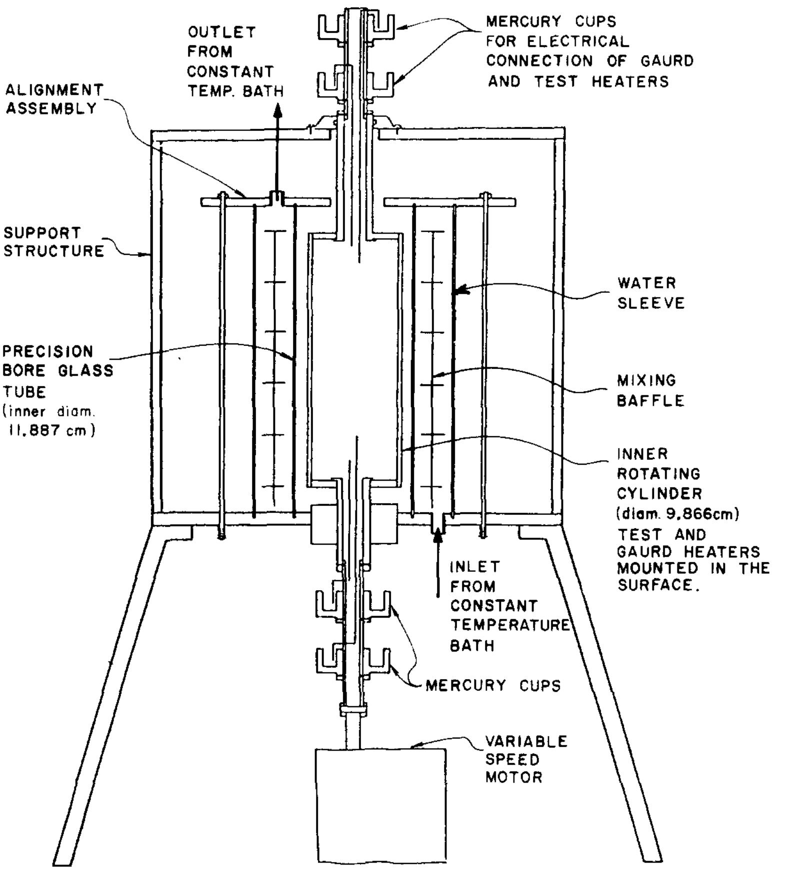

Figure 1. The experimental apparatus.

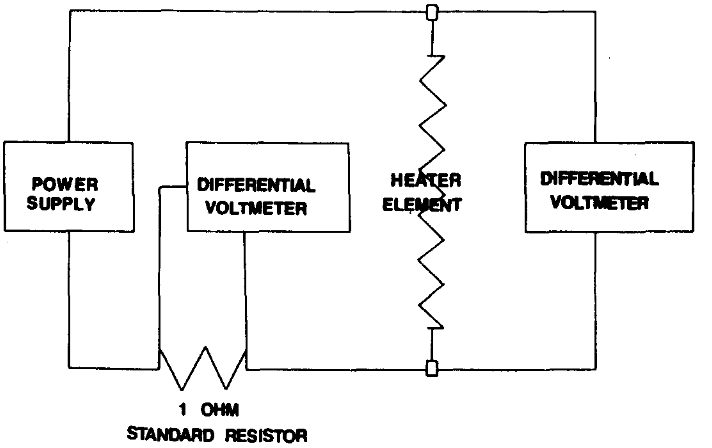

Figure 2. The electrical circuit and instruments used for the generation and measurement of heat flux.

the test heater, a precision DC power supply, Powers Designs model 605 was used. A Hewlett–Packard model 6268B power supply was used for the guard heating, The resulting voltage applied to the test and gaurd heaters was measured at the mercury cups. The electrical current flowing through the test heater was evaluated by measuring the voltage drop across a 1 Ω standard resistor placed in the circuit. Two John Fluke differential voltmeters, model 887 AB, were used to measure the voltage across the test heater and the standard resistor.

The desired heat flux across the gap between the two concentric cylinders was attained (approximately) by applying a predetermined voltage across the test heater. For all the experiments, a heat flux identical to that from the test heater was used for the guard heaters. Two different levels of heat transfer were used for different sets of experiments which resulted in temperature differences of about 2°C and 6°C, respectively, across the gap between the two concentric cylinders.

The experiments were performed with suspensions of polystyrene latex spheres (supplied by Diamond Shamrock Chemical Company) suspended in a Union Carbide lubricant, Ucon Oil HB-280-XY-23. The volume fraction of particles was varied from zero to 0.25. Three different sizes of spheres were used in the experiments; 34.6 μm in diameter with a standard deviation of 2.8 μm, 53.4 μm with a standard deviation of 3.8 μm and 122 μm with standard deviation equal to 4.7 μm. The use of different size particles was motivated primarily by the desire to extend the range of attainable values of Peclet number (proportional to the particle radius squared), a ploy necessitated by the fact that the range of allowable shear rates was limited by the onset of Taylor instability in the flow for $\gamma \geq \gamma_{crit}$, see Chandrasakhar (1961), and by mechanical unsteadiness of the Couette device for γ smaller than about 2.5 sec^{-1}. The suspending Ucon oil is Newtonian with a viscosity of about 1.34 poise and a density of 1.05 gm cm^{-3} at 26°C, the average temperature within the suspensions during most of the experiments (Union Carbide 1971). At the same temperature, the thermal conductivity of the oil is of the order 6.8×10^{-4} cal cm^{-1} (°C)$^{-1}$ sec^{-1}, the heat capacity about 0.44 cal gm^{-1}, and the coefficient of thermal expansion approximately 8×10^{-4} (°C)$^{-1}$.

Prior to any experiments, the suspension was carefully "conditioned". After adding the particles to the Ucon oil, the suspension was stirred vigorously by hand until it appeared to be homogeneous visually. It was then stirred continuously for an additional day with a magnetic stirrer before being transferred to the Couette flow apparatus. Finally, before any data were taken, the suspension was conditioned within the apparatus for 8–12 hr at the desired average temperature and at a slow rotational speed. A slight and reproducible increase in the water temperature was observed with increasing rotational speed of the inner cylinder, testifying to the effectiveness of the insulation of the system.

4. GOVERNING EQUATIONS FOR DATA ANALYSIS

An exact description of the temperature and velocity distributions in the Couette apparatus would be very difficult to obtain. The equations describing the velocity and temperature distributions are coupled since there are variations in the density and viscosity due to the nonuniform temperature distribution. Deviations from simple shear flow as a result of the curvature of the apparatus, the establishment of natural convection, the action of viscous dissipation, and the existence of temperature induced variations of material properties will all generally affect the temperature distribution. Furthermore, it is well known that the suspension is strictly Newtonian only as long as the concentration of the dispersed phase is low and the particle Reynolds number, describing the disturbance flow near the particles, is small (Jeffrey & Acrivos 1976). In addition, for a suspension undergoing nonuniform bulk flow, there can be flow-induced nonuniformity in the particle concentration across the gap; for example, particle-free (skimming) layers near the apparatus boundaries. Finally, for sufficiently large values of the angular velocity of the inner cylinder, the flow may become unstable, leading to a Taylor vortex structure in which the dominant mechanism for heat transfer across the gap is convection rather than conduction (Taylor 1923). Obviously, any of these effects would complicate the interpretation of experimental data considerably, and, in some cases, make the determination of an effective thermal conductivity impossible. Thus, both the experimental apparatus and the operating conditions for the experiments were designed carefully to minimize these various complications.

Detailed considerations of all of the effects listed above can be found in Chung (1980). In the interest of brevity, we shall simply summarize the results of these studies as they pertain to the present experiments. First, it may be anticipated that the effects of viscous dissipation will be small since the Brinkman number, a measure of the relative importance of viscous dissipation to heat conduction, was typically less than 10^{-4} for the experimental conditions employed. Second, an order of magnitude estimate can be made of the importance of the natural convection effects produced by the temperature differential across the gap of the Couette device, using the analysis of Batchelor (1954) which pertains strictly to natural convection between two vertical plane boundaries. In the present experiments, natural convection effects can be neglected provided: first, that the additional shear rate produced by the natural convection flow is small compared to that imposed by the rotation of the inner cylinder; second, that the heat flux across the gap due to natural convection is small relative to that by "conduction"; and, third, that any changes in the temperature profile in the test region (i.e. in the central part of our device) are negligible. All of these conditions can be met by making the temperature differential across the gap sufficiently small. In the experiments reported here, the maximum ΔT was 6°C, and thus the maximum Raleigh number was approximately 2.3×10^3. Our analysis using Batchelor's solutions shows that the temperature is unaffected over all but the final 2–3 cm at the top and bottom of the gap for this Rayleigh number—i.e. well away from the test region. Further, the maximum shear rate due to natural convection is approximately 0.28 $\sec^{-1}$ at the walls and this is less than 10% of the imposed shear rate which has a minimum value of 2.6 $\sec^{-1}$ and ranged up to approximately 40 $\sec^{-1}$ at the highest rotation rates. Finally, the maximum heat flux due to natural convection was 1.5×10^{-4} cal cm^{-2} sec^{-1} for $\Delta T = 6$°C, while the corresponding bulk heat flux due to conduction was approximately 10^{-2} cal cm^{-2} sec^{-1}. It may also be noted that the quantity of primary interest here is the contribution to the bulk conductive heat flux due to local convection effects at the particulate level. This quantity is determined as a difference between the measured total heat flux with shearing motion present and the measured heat flux across the same suspension in the absence of any imposed shear. With the same ΔT in both cases, it may be expected that this difference will be less sensitive to contributions of natural convection than either of the individual rates of heat transfer.

The instability of the shear flow at large angular velocities of the inner cylinder can, of course, be eliminated by operating the Couette device at angular velocities below that for the onset of instability. Indeed, Taylor (1923) showed that the flow is stable to arbitrary infinitesimal disturbances provided only that the characteristic Taylor number satisfies the condition

$$\mathrm{Ta} \leq 0.0525.$$

For the Ucon oil used in the experiments, this condition corresponds to a maximum rotational speed of about 10 rad s^{-1} prior to the onset of instability of the Ucon oil (figures 3 and 4). These results will be discussed in more detail in the next section. It was observed that the thermal conductivity increased rapidly, for a ΔT of 6°C, above rotational speeds of approximately 8.5 rad s^{-1} and, for a ΔT of 2°C, above rotational speeds of approximately 9 rad s^{-1}. In the experiments, the rotational speed was limited to approximately 8 rad s^{-1} in order to avoid the possibility of instability. The rheological properties of suspensions are influenced by the concentration, shape and size distribution of the particles, the presence of electrical charges, Brownian motion if the particles are very small, and the type of flow being experienced. As a result, suspensions in general are non-Newtonian. For this reason, the viscosity of the suspensions used was measured in a Stormer Viscometer, a rotating cup device capable of creating rotational shear flows similar to those of the Couette flow device used in this study. For shear rates up to approximately 40 s^{-1}, with a corresponding Reynolds number of not more than 10^{-3}, the effective viscosity was independent of both the shear rate and the particle size at any of the concentrations used here. These results are in agreement with the experimental observations of Krieger (1972). The presence of inhomogeneities in the particle concentration across the gap can result from lateral migration due to the presence of weak inertial effects; see, for example, Ho & Leal (1974), or Segré & Silberberg (1962). Since the typical experiments were run for several hours in order to allow the system to attain thermal equilibrium, it is important to verify that significant migration would not occur in that time period. Ho & Leal (1974) have shown that the lateral velocity for a single sphere due to inertia is proportional to $0(\mathrm{Re}\cdot\kappa^2)$ where Re is the particle Reynolds number $\mathrm{Re} \equiv Ga^2/\nu$, and κ is the ratio of the particle radius to the gap size. In our experiments, the maximum Re was $0(10^{-3})$ while the largest value of κ was $0(10^{-2})$. Thus, in a typical period of three hours, a particle could migrate no more than 10^{-3} cm, which is only 0.01% of the gap width, and a significant degree of nonuniformity could not be established in our experiments via the migration mechanism. The other source of nonuniform particle concentrations is the simple geomeric exclusion effect, which leads to a particle depleted region within one radius of the walls. In our experiments, the maximum particle size was only approximately 0.01 of the gap width, as already noted, and a simple analysis (Chung 1980) shows that the wall exclusion effect could not significantly affect the measured flow contribution to the effective conductivity.

Finally, the viscosity of the suspending fluid is strongly dependent on the temperature. For temperature differences across the gap of the Couette device of the order of 6°C, as existed in some of our experiments, the viscosity of the suspending fluid will vary in the gap by as much as 32%, and this will be reflected in a similar variation in viscosity for the suspension. The effective thermal conductivity of the suspension will also vary across the gap, both because the thermal conductivity of the suspending fluid depends on temperature and because viscosity variations and profile curvature will cause the velocity gradient (and thus the local Peclet number) to vary across the gap (see [1] where the dependence of k_s on Pe is demonstrated for a dilute suspension). The thermal conductivity of the Ucon oil which we used is relatively insensitive to temperature variations of 2–6°C which existed in our experiments. This simplifies the problem somewhat, but the temperature variation of the viscosity, and the resulting nonuniformity of the effective conductivity due to variation of the local Peclet number across the gap is not so obviously neglected. Surprisingly, however, a detailed analysis of the problem, Chung (1980), shows that the results obtained by completely neglecting the viscosity variations in data analysis are almost identical to the "exact" results which are obtained when they are included.

For evaluation of the average effective thermal conductivity in the gap of the Couette device, it is therefore sufficient to use the simple analysis in which the particle concentration, the viscosity and the thermal conductivity are all assumed to be constant. This approach is advantageous for presentation here, because it is very simple and the main elements of the theory and experiments are not obscured by algebraic complexity which add little to understanding.

Thus, adopting a cylindrical coordinate system with the axis of the cylinders of the Couette device as the z-axis, the equation of motion in the Θ-direction for a fluid of constant viscosity is

$$\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rV_\Theta)\right] = 0. \qquad [3]$$

The inner cylinder of the Couette device rotates at an angular speed of ω_1 and the outer cylinder is stationary. The boundary conditions are therefore:

$$V_\Theta(R_1) = \omega_1 R_1, \quad V_\Theta(R_2) = 0, \qquad [4]$$

where R_1 and R_2 are the radii of the inner and outer cylinders, respectively. The solution to this simple boundary value problem is the well-known Couette flow profile

$$V_\Theta(r) = \frac{\omega_1}{K^2 - 1}\left(\frac{K^2 R_1^2}{r} - r\right) \qquad [5]$$

where $K = R_2/R_1$.

The thermal energy equation for the suspension is

$$\bar{k}_s\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}T}{\mathrm{d}r}\right)\right] = 0 \qquad [6]$$

where $\bar{k}_s$ is the effective thermal conductivity of the suspension (assumed to be independent of r). For the outer glass cylinder wall, the thermal energy equation is

$$k_g\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}T_g}{\mathrm{d}r}\right)\right] = 0 \qquad [7]$$

where k_g is the thermal conductivity of the glass cylinder and T_g is the temperature distribution within the cylinder. The Couette device is subjected to a constant heat flux at the surface of the inner cylinder and held at a constant temperature at the outer surface of the outer cylinder. Thus, the boundary conditions are

$$\bar{k}_s\frac{\mathrm{d}T}{\mathrm{d}R}\bigg|_{r=R_1} = -q = \text{constant}$$

$$\bar{k}_s\frac{\mathrm{d}T}{\mathrm{d}R} = k_g\frac{\mathrm{d}T_g}{\mathrm{d}r}\bigg|_{r=R_2} = -\frac{q}{K} \qquad [8]$$

$$T(R_2) = T_g(R_2), \quad T_g(R_2 + \delta R) = T_w = \text{constant}$$

where T_w is the fixed temperature of the conditioning water at the outer surface of the glass cylinder and δR is the thickness of the wall of the glass cylinder. T can be obtained readily by solving [6] and [7] simultaneously:

$$T = \frac{qR_1}{\bar{k}_s}\ln\frac{R_2}{r} + \frac{qR_1}{k_g}\ln\frac{R_2 + \delta R}{R_2} + T_w. \qquad [9]$$

The effective thermal conductivity $\bar{k}_s$ can therefore be obtained from [9] by measuring the temperature $T(R_1)$ at the surface of the inner cylinder. The result for $\bar{k}_s$ is

$$\bar{k}_s = \frac{qR_1 \ln K}{T(R_1) - T_w - \frac{qR_1}{k_g}\ln\frac{R_2 + \delta R}{R_2}}. \qquad [10]$$

In the experiments, the heat flux, q, was generated by applying an electrical voltage E and current I through the test heater. The heat flux, q, per unit area of the test heater is then given by

$$q = \frac{EI}{2\pi R_1 L f} \qquad [11]$$

where L is the length of the test heater and f is the conversion factor:

$$1 \text{ cal s}^{-1} = f \text{ watts} = 4.184 \text{ watts}.$$

The temperature at the surface of the test heater, $T(R_1)$, was determined by measuring the electrical resistance of the heater, using the empirically determined relationship between the resistance and temperature of the wire

$$T(R_1) = 12.46\frac{E}{I} - 195.4. \qquad [12]$$

Substituting [11] and [12] together with the known geometric parameters of the system into [10] yields

$$\bar{k}_s = \frac{7.01 \times 10^{-4} EI}{12.46\frac{E}{I} - 195.4 - T_w - 3.731 \times 10^{-2} EI}. \qquad [13]$$

Here, $\bar{k}_s$ is the effective thermal conductivity in cal s^{-1} cm^{-2} (°C)$^{-1}$, E is the electrical voltage in volts, I is the electrical current in amperes and T_w is the temperature of the conditioning water in °C.

Our primary objective in this study is to determine the functional dependence of the effective thermal conductivity $\bar{k}_s$ on the Peclet number relevant to the particle,

$$\text{Pe}_1 = \frac{\rho C_p}{k_1} G\bar{a}^2$$

where ρ is the density of the fluid, C_p is the heat capacity, $\bar{a}$ is the mean particle radius and G is the local shear rate experienced by the suspended particles. The detailed analysis of Chung (1980), referred to above, which includes the radial dependence of the shear rate, $G(r)$ due to curvature effects from the flow device and to the temperature dependence of the viscosity, shows that it is sufficient to consider $\bar{k}_s$ as a function of the average Peclet number, $\overline{\text{Pe}}$, defined using the radially averaged shear rate, i.e.

$$\overline{\text{Pe}}_1 = \frac{\rho C_p}{k_1} \bar{G}a^2 \qquad [14]$$

where

$$\bar{G} = \frac{\left[\int_{R_1}^{KR_1} G(r)\,\mathrm{d}r\right]}{\left[\int_{R_1}^{KR_1} \mathrm{d}r\right]} = \frac{-\omega_1}{K-1} = -4.94\,\omega_1\,(\text{s}^{-1}) \qquad [15]$$

to obtain results that can be compared directly to [1]. It may be noted that the average shear rate $\bar{G}$ is identical to the shear rate in a Newtonian fluid between two infinite parallel plates which are separated by a distance $(R_2 - R_1)$, with one plate moving at a speed of $\omega_1 R_1$ and the second plate stationary.

5. ERROR ANALYSIS

The degree of accuracy in the measurement of thermal conductivity can be established by considering [13] used in the analysis of the data. The error $\delta\bar{k}_s$ in the measured thermal conductivity caused by errors δE, δI and ωT_w in the measurements of the voltage, current and conditioning water temperature is given, to first-order, by the following equation

$$|\delta \bar{k}_s| = \left|\frac{\partial \bar{k}_s}{\partial E}\,\delta E\right| + \left|\frac{\partial \bar{k}_s}{\partial I}\,\delta I\right| + \left|\frac{\partial \bar{k}_s}{\partial T_w}\,\delta T_w\right|. \qquad [16]$$

The partial differentials in [16] may be obtained from [13]. When they are evaluated at the typical experimental conditions of $E \approx 5.64$ volts, $I \approx 0.315$ ampere and $T_w \approx 25.00$°C, corresponding to $\Delta T \approx 2$°C [16] then becomes

$$|\delta \bar{k}_s| = (6.73 \times 10^{-3}|\delta E| + 1.24 \times 10^{-1}|\delta I|$$
$$+ 1.78 \times 10^{-4}|\delta T_w|)\ \mathrm{cal\ s^{-1}\ cm^{-1}\ {}^\circ C^{-1}}. \qquad [17]$$

The electronics and measuring systems used in this study (see section III) have typical associated errors

$$|\delta E| < 0.00001 \text{ volt}, |\delta I| < 0.0001 \text{ amp and } |\delta T_w| < 0.01^\circ\text{C}.$$

If a conservative value of $\bar{k}_s \sim 6.8 \times 10^{-4}$ cal sec^{-1} cm^{-1} °C^{-1} is used (this is approximately equal to the measured thermal conductivity of Ucon Oil), we obtain the error estimates for $\bar{k}_s$ in table 1. Thus, the expected percentage error in $\bar{k}_s$ is:

$$\frac{|\delta \bar{k}_s|}{\bar{k}_s} < 0.54\%. \qquad [18]$$

For a $\Delta T \approx 6$°C, the expected percentage error in $\bar{k}_s$, for errors of the same magnitude, is slightly less than 0.54%.

The uncertainty in the measured T_w gives rise to the largest share of the error in $\bar{k}_s$. This seemingly negligible error in $\bar{k}_s$ is actually comparable to the predicted enhancement in conductivity [1] due to a simple shear when both the local Peclet number and the volume concentration of the dispersed phase are small. It is for this reason that it was necessary to use such high precision instrumentation in the experiment, and this is also the reason why McMillen's early work (1976) was inconclusive.

6. EXPERIMENTAL RESULTS

(a) *The thermal conductivity of Ucon Oil HB-280-XY-23*

The thermal conductivity of the Ucon Oil HB-280-XY-23, used as the suspending fluid in this study, is known as a function of temperature (Union Carbide 1971). If the rotational Couette device operated in an ideal manner for the purposes of measuring thermal conductivity, the value of k_1 determined for the oil alone, should be independent of the rotational speed and equal to the known value. However, thermal dissipation and other secondary effects are unavoidable in the Couette device. An estimate of the magnitude of such effects in the actual

Table 1. Error estimates

Error	$\frac{\lvert\delta \bar{k}_s\rvert}{\bar{k}_s} \times 100$ (%)
$\lvert\delta E\rvert < 0.001$	< 0.099
$\lvert\delta I\rvert < 0.00001$	< 0.18
$\lvert\delta T_w\rvert < 0.01$	< 0.26

experiments can be made by applying the formula [10] to determine apparent values of the thermal conductivity for Ucon Oil alone. In the absence of secondary effects, the thermal conductivity determined in this way should be independent of rotation speed, and independent of the temperature difference across the gap (though not of the average temperature of the oil).

Figures 3 and 4 are plots of the measured thermal conductivity of the Ucon Oil alone, i.e. k_1, at different rotational speeds ω_1 of the inner cylinder. The temperature differences, ΔT, across the annular gap between the cylinders were approximately 2°C and 6°C. With a ΔT of 2°C, it is evident that the measured thermal conductivity of the Ucon Oil increases slightly at a rate which is approximately proportional to the square of the angular velocity of the inner cylinder for low rotational speeds. This rate of increase suggests strongly that the increase was due to viscous dissipation, and this conjecture is supported by the fact that the proportionality

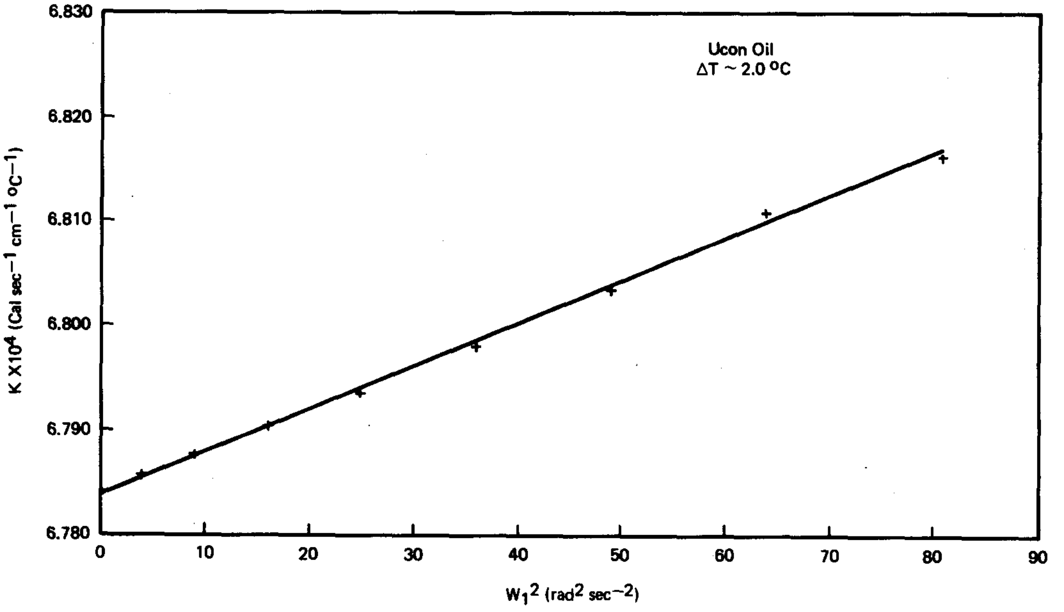

Figure 3.

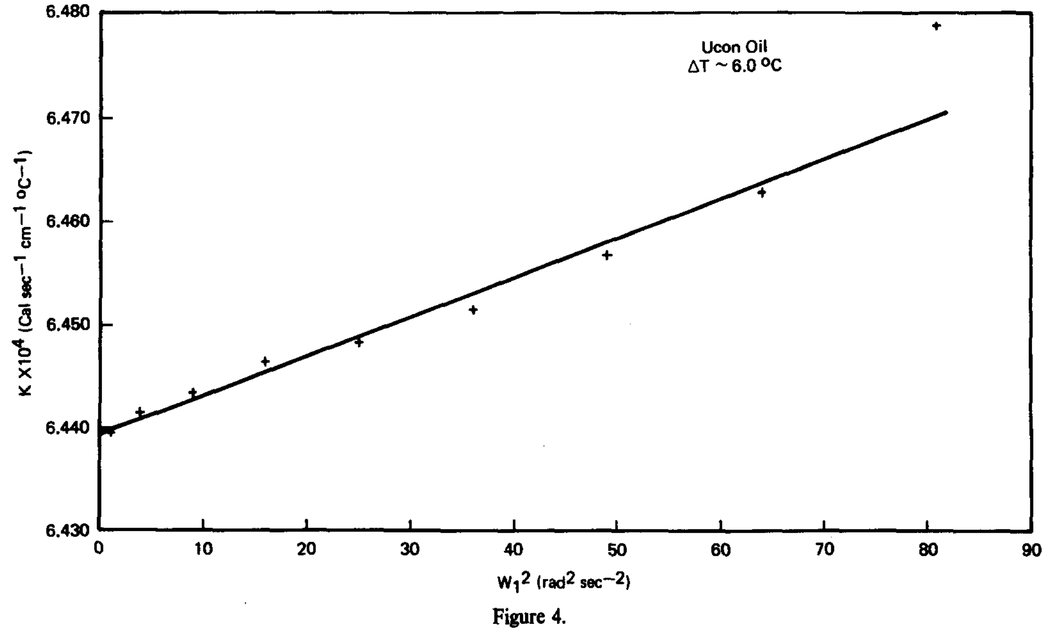

Figure 4.

Figures 3–4. Measured thermal conductivity of Ucon Oil at various rotational speeds.

constant between k_1 and ω_2 is very close to that which can be predicted theoretically based on this assumption. At rotational speeds above $9\,\mathrm{rad\,s^{-1}}$, the apparent k_1 obtained from [10] increased more rapidly with rotational speed, possibly signifying the onset of instability of the flow.† Very similar behavior was again observed for a ΔT of approximately 6°C. In this case the increase in k_1 was substantial at rotational speeds above $8.5\,\mathrm{rad\,s^{-1}}$. For all subsequent experiments, the rotational speed was limited to $9\,\mathrm{rad\,s^{-1}}$ for $\Delta T \sim 2$°C and to $8\,\mathrm{rad\,s^{-1}}$ for $\Delta t \sim 6$°C.

Even the slight increase in the measured thermal conductivity with increasing rotational speed at the lower rotation rates cannot be neglected in the analysis of suspension data. The fractional increase in thermal conductivity due to the action of shear on the particles is expected to be of $0(\varphi\,\mathrm{Pe}^{3/2})$ and thus very small for small φ and Pe. Indeed, the small increase in thermal conductivity shown in figures 3 and 4 for $\omega_1 < 8$–$9\,\mathrm{rad\,s^{-1}}$ was only slightly smaller than the predicted increase in thermal conductivity due to the shear flow in very dilute suspensions. The method used in data analysis for the suspension to account for the apparent shear dependence of the conductivity of the Ucon Oil will be discussed later in this section.

It may also be noted that the measured thermal conductivity was slightly different for the two temperature differences used. At $\Delta T \sim 2$°C and no rotation, the measured thermal conductivity was $6.784 \times 10^{-4}\,\mathrm{cal\,s^{-1}\,cm^{-1}\,°C^{-1}}$, while the corresponding value at $\Delta T \sim 6$°C was $6.437 \times 10^{-4}\,\mathrm{cal\,s^{-1}\,cm^{-1}\,°C^{-1}}$. The slight difference in the two values may be due to the higher average temperature in the Ucon Oil when ΔT was larger. The thermal conductivity of the Ucon Oil does decrease with increasing temperature (Union Carbide 1971), but the data available (Union Carbide 1971) are not sufficiently accurate to obtain a quantitative verification of this hypothesis for these small changes in temperature.

(b) *The thermal conductivity of the polystryene latex spheres*

For suspensions of rigid spherical particles, the increase in thermal conductivity at low local Peclet number is predicted by Leal (1973) to be a weak function of the thermal conductivity of the particles. This point can be demonstrated readily from [1]. In particular, it can be seen from [1] that the fractional increase in thermal conductivity in a simple shear flow is

$$\frac{\Delta k_s}{k_1} = \left[1.176\frac{(k_2-k_1)^2}{(k_2+2k_1)^2} + 3.0 - 0.14\frac{(k_2-k_1)}{(k_2+2k_1)^2}\right]\Phi\,\mathrm{Pe}_1^{3/2} \qquad [19]$$

where Δk_s is the increase in thermal conductivity due to shear.

Limiting values for this expression are:

$$\frac{\Delta k_s}{k_1} = 3.364\Phi\,\mathrm{Pe}_1^{3/2} \text{ when } \frac{k_2}{k_1}\to 0 \qquad [20]$$

$$\frac{\Delta k_s}{k_1} = 3.0\Phi\,\mathrm{Pe}_1^{3/2} \text{ when } \frac{k_2}{k_1} = 1 \qquad [21]$$

and

$$\frac{\Delta k_s}{k_1} = 4.036\Phi\,\mathrm{Pe}_1^{3/2} \text{ when } \frac{k_2}{k_1}\to\infty. \qquad [22]$$

It is thus evident that it is not critical to have an exact value of k_2 as long as the ratio (k_2/k_1)

†The critical Taylor number for an isothermal system with Ucon Oil would occur at an angular velocity of approximately 10 radians/sec. However, the *apparent* critical values from our apparatus appear to decrease slightly as the temperature difference across the gas is increased.

is of 0(1). This requirement will be satisfied with the highly conductive Ucon Oil for any reasonable estimate of k_2. On the other hand, if the value of k_2 were known, [1] would provide an exact theoretical prediction for either a sheared or stationary suspension of neutrally buoyant spheres.

A number of techniques exist for determination of the thermal properties of solids and liquids. It is, however, difficult to measure the thermal properties of an aggregate of small solid particles. Thus, instead of measuring the thermal conductivity of the particles directly, the thermal conductivity was deduced indirectly using measured values of the thermal conductivity of *stationary* suspensions of polystyrene latex spheres at different concentrations, and Maxwell's theoretical predictions for the same case

$$\frac{k_{so}}{k_1} = 1 + 3\frac{k_2 - k_1}{k_2 + 2k_1}\varphi + 0(\varphi^2) \qquad [23]$$

where k_{so} is the thermal conductivity of a stationary suspension with volume fraction φ.

Figures 5 and 6 are plots of the measured values of the thermal conductivity vs concentration for the stationary suspensions. At $\Delta T \sim 6°C$ only the particles with $\bar{a} = 26.7$ microns were used. At $\Delta T \sim 2°C$, all three different size particles were used. Within experimental accuracy, the k_{so} of suspensions prepared from particles of different sizes, as shown in figure 6, where essentially the same. Both plots 5 and 6 are virtually linear to a volume fraction of 0.25.

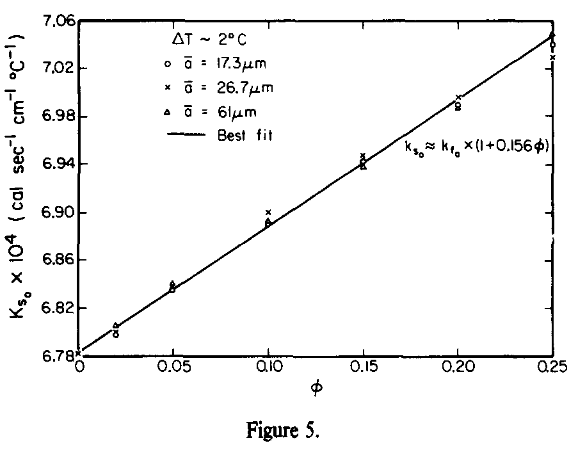

Figure 5.

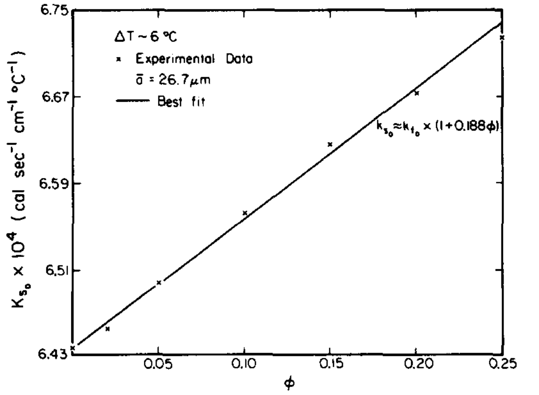

Figure 6.

Figures 5–6. Measured thermal conductivity of stationary suspensions at different volume fractions.

When $\Delta T \sim 6°C$, a best fit of the data was given by

$$k_{so} \approx (6.437 \times 10^{-4})(1 + 0.188\varphi)\ \text{cal s}^{-1}\ \text{cm}^{-1}\ °\text{C}^{-1}. \qquad [24]$$

Hence, applying [23], k_2 was estimated as

$$k_2 = 7.728 \times 10^{-4}\ \text{cal s}^{-1}\ \text{cm}^{-1}\ °\text{C}^{-1}. \qquad [25]$$

When $\Delta T \sim 2°C$, on the other hand, the thermal conductivity k_{so} was given by

$$k_{so} \approx (6.78 \times 10^{-4})(1 + 0.156\varphi)\ \text{cal s}^{-1}\ \text{cm}^{-1}\ °\text{C}^{-1} \qquad [26]$$

By applying [23] again,

$$k_2 = 7.900 \times 10^{-4}\ \text{cal s}^{-1}\ \text{cm}^{-1}\ °\text{C}^{-1}. \qquad [27]$$

The dependence of k_2 on temperature is thus seen to be weaker than that of the Ucon Oil.

Jeffrey (1974) obtained theoretical second-order corrections to Maxwell's prediction:

$$\frac{k_{so}}{k_1} = 1 + 3\gamma\phi + \phi^2\left[3\gamma^2 + \frac{3}{4}\gamma^3 + \frac{9}{16}\gamma^3\frac{\alpha+2}{2\alpha+3} + \frac{3\gamma^4}{2^6} + \cdots\right] \qquad [28]$$

where $\alpha = (k_2/k_1)$ and $\gamma = (\alpha - 1)/(\alpha + 2)$. If k_2 given in [25] is used for this second-order correction, the following relation results

$$\frac{k_{so}}{k_1} = 1 + 0.188\varphi + 0.012\varphi^2 + \cdots. \qquad [29]$$

Similarly, for the k_2 given in [27]

$$\frac{k_{so}}{k_1} = 1 + 0.156\varphi + 0.0082\varphi^2 + \cdots. \qquad [30]$$

The second-order corrections (in terms of concentration, φ) are therefore negligible for the concentrations used, as indicated by the plots in figures 5 and 6.

(c) *The effective conductivity of sheared suspensions as a function of local Peclet number*

(*i*) *Data analysis.* In the measurement of the thermal conductivity of suspensions at various Peclet numbers, an apparent thermal conductivity, k_s^*, was first determined directly from [13] based on the measured values of E, I and T_w—i.e. the total heat input and temperature jump across the gap of the Couette device. This value, k_s^*, clearly reflects not only the modification in thermal conductivity of the suspension due to local shear effects, but also changes in the intrinsic thermal properties of the suspension (as predicted by Maxwell) and the spurious influence of any residual secondary effects that have not been eliminated by the precautions outlined in section 4. Equation [1] shows that the "direct" effect of shear is simply an additive "correction" to the Maxwell formula for the effective thermal conductivity of a stationary suspension. Thus, in the absence of any spurious secondary effects, k_s^* would equal the average conductivity of the suspension $\bar{k}_s$, and the microconvective contribution to the effective conductivity could be isolated by simply subtracting the predicted or measured heat flux in the stationary state from the measured local heat flux when the suspension is sheared. Unfortunately, however, the data of figures 3 and 4 indicate an apparent spurious dependence of the thermal conductivity on shear rate due to viscous dissipation effects, even for the suspending fluid alone. In the suspension then, it is evident that k_s^* will vary with shear rate both because of the microconvective contribution to the true effective conductivity $\bar{k}_s$ which it is desired to measure, and because of viscous dissipation effects. No rigorous means exist to separate these two effects. In the present study, we have therefore resorted to an empirical approach in which it is assumed that the shear-rate dependence of k_s^* due to viscous dissipation will have the same functional dependence on the rotational speed of the inner cylinder, ω_1, for the suspension as is exhibited for the suspending fluid alone. The contribution to the effective thermal conductivity due to the presence of the shearing flow Δk_s, was therefore estimated from the expression

$$\Delta k_s(\omega_1) = k_s^* - k_{so}(1 + m\omega_1^n) \qquad [31]$$

with the constants m and n obtained from the plots of figures 3 and 4 which pertain strictly to the suspending fluid alone. No direct means exists to check the assumptions inherent in [31]. However, we shall see that the "measured" dependence of Δk_s (calculated from [31]) on shear-rate (or ω_1) is identical to the predicted $\mathrm{Pe}_1^{3/2}$ dependence of Leal (1973), and this, in itself, constitutes strong evidence that the assumption is correct within measurable levels of the independent variables.

Substituting the values of k_1 and k_2 obtained earlier into Leal's (1973) theoretical prediction for Δk_s, we find

$$\Delta k_s = 3.00 k_1 \varphi \, \mathrm{Pe}_1^{3/2} \qquad [32]$$

for a dilute suspension.

As a test of the above expression, the relative augmentation in conductivity, $R = (\Delta k_s / k_1 \varphi)$, was calculated for each data point and plotted against the radially averaged Peclet number $\overline{\mathrm{Pe}}_1$. Figures 7–13 give the resulting plots at different volume fractions, φ, along with the theoretical prediction, [32].

The effect of any possible heat loss to the inside of the inner cylinder was not considered in section 6. Since the temperature of the inner cylinder was virtually constant throughout each set of experiments, any heat loss to the inside of the inner cylinder should have been independent of Peclet number. By presenting the experimental results in terms of R instead of the measured thermal conductivity, any effect of possible heat transfer to the inside of the inner cylinder will be minimal.

(*ii*) *Results and discussion.* The experimental data plotted in figures 7–13 were obtained over a range of Peclet numbers from about 0.01 to 1.3, by varying the rotational speed and the size of the dispersed particles. Data for all suspensions with the same volume fraction of the dispersed phase were plotted together.

Within experimental accuracy, there was no appreciable difference in the enhancement of conduction due to different sizes of the particles over the range of volume fraction from 0.01 to 0.25 used in the study. For the smallest particles with an average radius of 17.3 μm, the ratio between the annular gap width of the apparatus and the diameter of the particles was about 300. At this very large ratio the suspension is readily viewed as macroscopically homogeneous. For the largest particles, the same ratio was about 82. The lack of any discernible dependence of Δk_s on the particle size indicates that suspensions with these rather large particles could be still viewed as homogeneous within the present apparatus.

As mentioned earlier, the data were plotted in terms of the relative augmentation in thermal conductivity, R, as a function of radially averaged Peclet number $\overline{\mathrm{Pe}}_1$. The theoretical prediction for R is given by

$$R = R_1 = 3.0 \, \mathrm{Pe}_1^{3/2} \qquad (33)$$

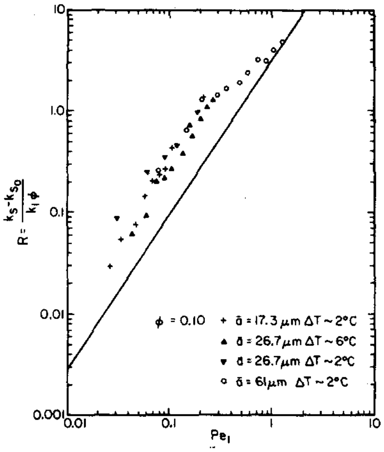

Figure 7.

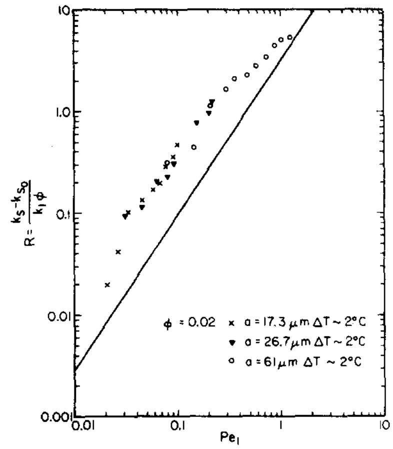

Figure 8.

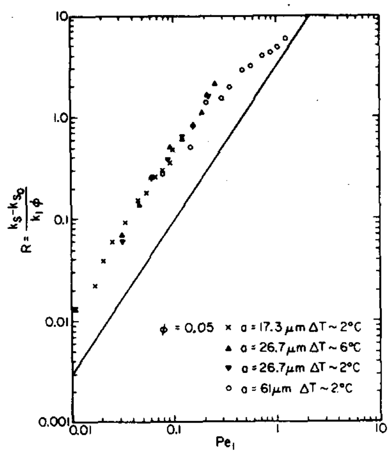

Figure 9.

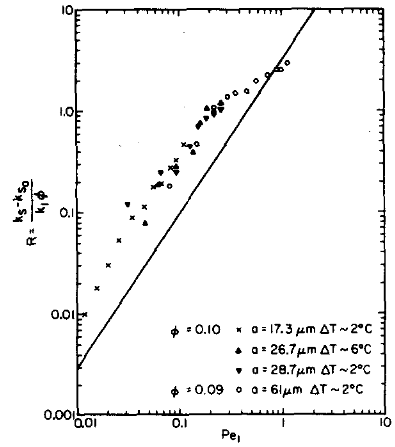

Figure 10.

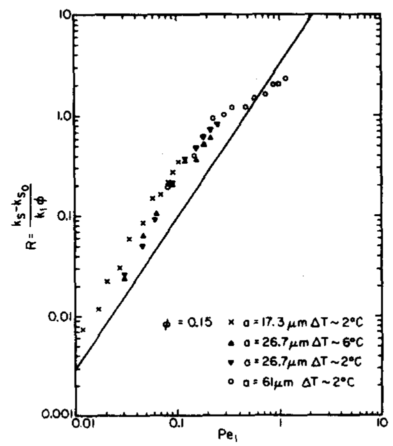

Figure 11.

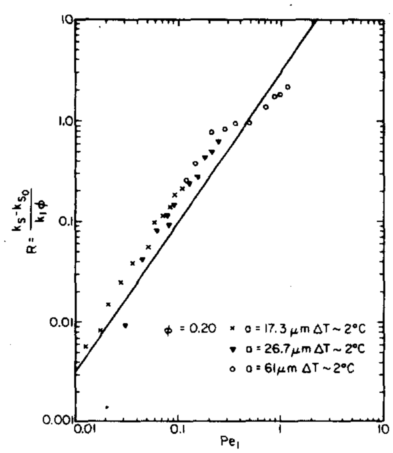

Figure 12.

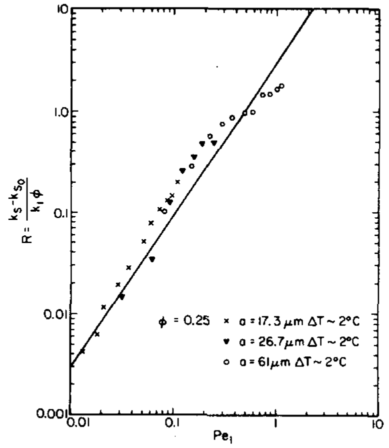

Figure 13.

Figures 7–13. Plots of relative augmentation in thermal conductivity R vs. $\overline{Pe}_1$ at fixed volume fractions.

To test this expression, each set of data was assumed to be correlated by the general expression

$$R = C\overline{Pe}_1^d$$

and the constants C and D were then calculated by a least squares regression analysis for each set of data. Tables 2–4 list the results of the calculation. For $\overline{Pe}_1$ up to 0.27, the exponent d varied from 1.21 to 1.91. However, the majority of values for different volume fractions were very close to the theoretical value of 1.5. On the other hand, it was found that the constants, C, departed substantially from the theoretical value of 3.0, even for small Pe and φ. It must be noted, however, that the constant C is very sensitive to the exponent d at small Pe. For example, in table 2, the "measured" $C = 29.9$ at $\varphi = 0.01$ was almost ten times the theoretical value. The exponent d in this case was 1.91. However, if the empirical value of R (either measured or calculated from the measured C and d) and the theoretically predicted R_1 (equal to 3.0 $Pe_1^{3/2}$) are compared at Peclet numbers relevant to this case, we obtain at

$$Pe_1 = 0.01, \quad \frac{R}{R_1} = \frac{29.9(0.01)^{1.91}}{3.0(0.01)^{1.5}} = 1.51,$$

while at

$$Pe_1 = 0.1, \quad \frac{R}{R_1} = \frac{29.9(0.1)^{1.91}}{3.0(0.1)^{1.5}} = 3.88.$$

Thus, the predicted augmentation for R differs less from the measured (or empirical) correlation than it might first appear.

When both the volume fraction and local Peclet number were small, the measured R was always larger than the theoretically predicted value. The difference between the measured R and the predicted value, R_1, first increased with increasing volume fraction φ. When φ was increased further, however, the difference between the theoretical and experimental values began to decrease. In fact, the measured R at higher volume fractions fortuitously agreed very well with the theoretical prediction for dilute suspensions over most of the range of Pe.

The deviation of the experimental data from the "predicted" correlation was also dependent on the Peclet number. It first increased with increasing $\overline{Pe}_1$, and then gradually decreased when $\overline{Pe}_1$ approached $O(1)$, for all volume fractions. The decrease in this deviation when $\overline{Pe}_1 \sim O(1)$

Table 2. Empirical correlations for relative augmentation $R = C\overline{\mathrm{Pe}}_1^d$ for $\bar{a} = 17.3\ \mu m$, $\Delta T \approx 2°C$

φ	Range of $\overline{\mathrm{Pe}}_1$	C	d
0.01	0.0270-0.102	29.9	1.91
0.02	0.0206-0.100	16.3	1.58
0.05	0.0113-0.100	21.3	1.64
0.10	0.0119-0.111	18.6	1.66
0.15	0.0122-0.103	20.4	1.78
0.20	0.0128-0.107	11.4	1.74
0.25	0.0132-0.109	12.1	1.84

$\Delta T = 2°C$

$\bar{a} = 17.3\ \mu m$

$R = C\overline{\mathrm{Pe}}_1^d$

Table 3(a). Empirical correlations for relative augmentation $R = C\overline{\mathrm{Pe}}_1^d$ for $\bar{a} = 26.7\ \mu m, \Delta T \approx 2°C$

φ	Range of $\overline{\mathrm{Pe}}_1$	C	d
0.01	0.0428-0.266	10.6	1.62
0.05	0.0310-0.248	17.8	1.56
0.10	0.0459-0.249	10.1	1.48
0.15	0.0310-0.214	8.12	1.61

$\Delta T \approx 2°C$

$\bar{a} = 26.7\ \mu m$

$R = C\overline{\mathrm{Pe}}_1^d$

Table 3(b). Empirical correlations for relative augmentation $R = C\overline{\mathrm{Pe}}_1^d$ for $\bar{a} = 26.7\ \mu m$, $\Delta T \approx 6°C$

φ	Range of Pe_1	C	d
0.01	0.0309-0.214	8.83	1.34
0.02	0.0309-0.214	9.86	1.40
0.05	0.0309-0.214	13.7	1.48
0.10	0.0309-0.250	5.91	1.21
0.15	0.0309-0.250	8.33	1.58
0.20	0.0309-0.214	5.72	1.57
0.25	0.0309-0.233	8.85	1.62

$\Delta T = 6°C$

$\bar{a} = 26.7\ \mu m$

$R = C\overline{\mathrm{Pe}}_1^d$

was more evident at the higher volume fractions. When $\overline{\mathrm{Pe}}_1 \sim 0(1)$, the power dependence on Peclet number also became much smaller, as shown in table 5. At the higher values used for $\overline{\mathrm{Pe}}_1$ and φ, the measured R eventually became smaller than the corresponding theoretical value from [33].

The fact that the difference between the measured and predicted relative augmentation in conductivity, R, increased with increasing volume fraction φ from 0.01 to 0.05 indicates that these low concentrations still did not approach the dilute limit necessary for application of the theory. Some of the difference between the measured and predicted R's may have been due also to non-negligible physicochemical interactions between the particles and the resulting formation of doublets and higher-order aggregates. The presence of such aggregates would lead to an effective average particle radius larger than a, and thus an increase in R due to the corresponding increase in the local Peclet number. However, we do not believe that this latter effect was very important. In the first place, visual examination of the suspension under a microscope showed a negligible number of doublets. Furthermore, it would be expected that a significant contribution of doublets to R would be accompanied by a dramatic decrease in the exponent d since nonspherical particles are known to augment the thermal conductivity at a rate proportional to Pe rather than $\mathrm{Pe}^{3/2}$ for single spheres. No significant decrease in d below

Table 4. Empirical correlations for relative augmentation $R = C\overline{\mathrm{Pe}}_1^d$ for $\bar{a} = 61\ \mu\mathrm{m}$, $\Delta T \approx 2°\mathrm{C}$

φ	Range of $\overline{\mathrm{Pe}}_1$	C	d
0.01	0.0786-0.476	5.41	1.12
	0.582-1.28	3.81	0.840
	0.786-1.28	4.06	0.957
0.02	0.0800-0.474	6.81	1.26
	0.579-1.28	4.63	0.886
	0.0800-1.28	4.96	1.06
0.05	0.0786-0.470	1.01	1.40
	0.568-1.26	4.83	0.748
	0.0786-1.26	5.44	1.04
0.09	0.0820-0.474	5.54	1.29
	0.572-1.17	2.66	0.555
	0.0820-1.17	3.11	0.955
0.15	0.0816-0.470	3.64	1.12
	0.568-1.16	2.09	0.638
	0.0816-1.16	2.33	0.861
0.20	0.0823-0.481	3.60	1.28
	0.566-1.17	1.87	0.754
	0.0823-1.17	2.13	0.987
0.25	0.0817-0.480	2.97	1.26
	0.582-1.06	1.69	0.871
	0.0817-1.06	1.88	1.016

$\Delta T = 2°\mathrm{C}$

$\bar{a} = 61\ \mu\mathrm{m}$

$R = C\overline{\mathrm{Pe}}_1^d$

1.5 was apparent in the data for $\overline{\mathrm{Pe}}_1 < 0.27$, even though the measured R is considerably larger than the predicted value.

A somewhat similar conclusion also applies for the dependence on Peclet number. The fact that the difference between the measured and predicted R's increased with increasing $\overline{\mathrm{Pe}}_1$ when $\overline{\mathrm{Pe}}_1 \sim 0(10^{-1})$ again seems to indicate that the Peclet numbers used in this study were not sufficiently small to verify the theory. Conversely, the available theory did not account for all possible interaction effects contributing to enhancement in conduction at Peclet numbers ranging up to approximately 1. The dependence on $\overline{\mathrm{Pe}}_1$ became gradually weaker at higher $\overline{\mathrm{Pe}}_1$. This agrees, at least partially, with the prediction by Nir & Acrivos (1976) that for large Pe_1 the variation in enhancement with Pe_1 is extremely weak. However, the largest $\overline{\mathrm{Pe}}_1$ used in this study was only of 0(1), and we obviously could not test Nir & Acrivos' prediction of a $0(\mathrm{Pe}_1^{1/11})$ enhancement to conduction at large Pe_1.

At all volume fractions and Peclet numbers used in this study, the observed enhancement in thermal conductivity was of the same order of magnitude as predicted by Leal's analysis for small φ and Pe_1 (1973). This result was unexpected at high Pe_1 and high volume concentrations of particles. However, it appears that the theory could be used for *order of magnitude estimations* of thermal conductivity of sheared suspensions over the whole range of conditions that are considered in this study, in spite of the fact that it obviously does not give a complete physical description of the transport behavior.

Better qualitative agreement between experimental observation and theoretical prediction might be achieved with more dilute suspensions and smaller Peclet numbers than those used in this study. However, the experimental accuracy required to detect the extremely small changes under these conditions precludes such a study in our apparatus.

The measured enhancement in thermal conductivity differed slightly at the two temperature differences, ΔT, used. At $\Delta T \sim 2°C$, however, the thermal conductivities measured were on the average not more than 3% larger than the corresponding thermal conductivities measured at $\Delta T \sim 6°C$.

7. CONCLUSION

The observed enhancement in the thermal conductivity in a sheared suspension in this study was always of the same order of magnitude as predicted by Leal's theory (1973). Though Leal's theory was limited to dilute suspensions, it agreed amazingly well with observed enhancements in conductivity even for moderately concentrated suspensions. However, it is evident from the detailed variations between the experimental observations and theoretical predictions with Pe and φ, that the theory does not completely describe the transport behavior for the whole range of Peclet numbers and volume concentrations used in this study. Surprisingly, the experimental observations indicated that volume fractions of $0(10^{-2})$ and Peclet numbers of $0(10^{-1})$ are not sufficiently small to approach the asymptotic theory for Pe$\to 0$, $\varphi \to 0$.

Observations on the heat transport behavior for Peclet numbers of 0(1) and higher concentrations may also be relevant to many problems in enhanced molecular diffusion. The lack of any theoretical analysis in these regimes makes the results obtained in this study particularly useful. It should be noted, however, that the empirical correlations obtained from this study are extremely sensitive to the Peclet number and should not be used for Peclet numbers much different from those for which the correlations were derived.

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